

QUANTIFYING UNCERTAINTIES IN THE STRUCTURAL RESPONSE OF SSME BLADES

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A probabilistic study on turbopump blades of Space Shuttle Main Engine (SSME) undertaken at NASA Lewis is reaching the three year mark. The ultimate objective of this study is to evaluate the effects of random variations, generally called uncertainties, in the geometry, material properties, loading and their probabilistic combinations on the structural response of the blade. The results obtained on geometric and material properties uncertainties demonstrate that a methodology using probabilistic structural analysis methods appears to be a powerful and unique approach to quantify uncertainties.

The application of this methodology can be extended to quantify the effects of random variations for other structural components also. Therefore, the development of the methodology has been discussed here in a rather generalized manner. This methodology consists of the following steps:

- (1) The geometry and material properties can be randomly perturbed to simulate the realistic uncertainties. The simulation can be conducted using random numbers and using advanced perturbation techniques such as Monte Carlo's simulation. The random numbers can be selected to have any given probabilistic distributions which have known statistical properties. In this study, the normal distribution was selected with known mean and standard deviation.
- (2) An analysis technique such as finite elements method can be used to estimate the structural response and provide means for discrete perturbations. The blade geometry and material properties were perturbed on the node and the element basis, respectively.
- (3) Statistical experiment designs such as full fractional design can be used to determine the effects of study variables. The advantage of using this design is that it is cost effective and provides the estimates of the effects of individual study variables and their interactions.
- (4) The effects of all the study variables and their interactions can be evaluated for their significance using statistical tests. Further the probabilistic models can be developed to predict a mean response for given variations in the study variables. Probability distributions for response variables can also be developed for estimates of their range of variations. For this study, t-test, F-test, and χ^2 -test were used.

To quantify the uncertainties associated with the geometry and material properties of a SSME turbopump blade, a computer code known as STAEBL was used. A finite element model of the blade used 80 triangular shell elements with 55 nodes and five degrees of freedom per node. The whole study was simulated on the computer and no real experiments were conducted. The structural response has been evaluated in terms of three variables which are natural frequencies, root (maximum) stress, and blade tip displacements.

The nodal coordinates (x , y , z) of the finite element mesh were perturbed to simulate the geometric uncertainties. The numbers of the material property matrix were perturbed for each element to simulate the material properties uncertainties. The perturbations were generated by a random number generator with preselected means and standard deviations. The magnitudes of means and standard deviations for perturbations of both geometry and material properties, in this study, were taken as ten percent or less of the original values. These magnitudes were selected based on previous experimental results and experience.

The results of the study indicate that only the geometric uncertainties have significant effects on the response. Uncertainties in material properties have insignificant effects. Also, the material properties interaction effects, which were created by variation in both material properties and geometry together have been found to have insignificant effects. A set of probabilistic models has been developed to predict the structural response for any given variations in geometry and material properties. Separate probabilistic models for only geometry variations have also been developed. Statistical tests indicate that these models are good fits.

OBJECTIVE

CONDUCT PROBABILISTIC STRUCTURAL ANALYSIS
OF SSME BLADES TO QUANTIFY UNCERTAINTIES
ASSOCIATED WITH:

GEOMETRY

MATERIAL PROPERTIES

WHY?

EVALUATE TOLERANCE LIMITS

- HIGHER TOLERANCE
- LOWER COST

BLADE GEOMETRY

SSME HIGH-PRESSURE FUEL TURBOPUMP
1st STAGE TURBINE BLADE

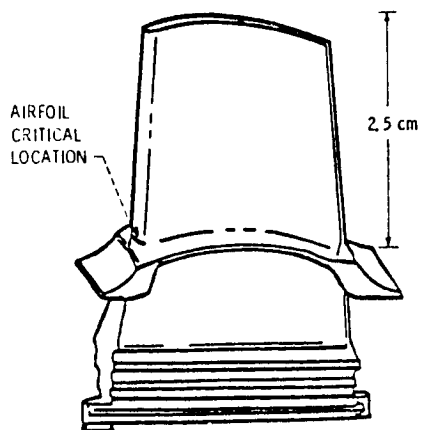


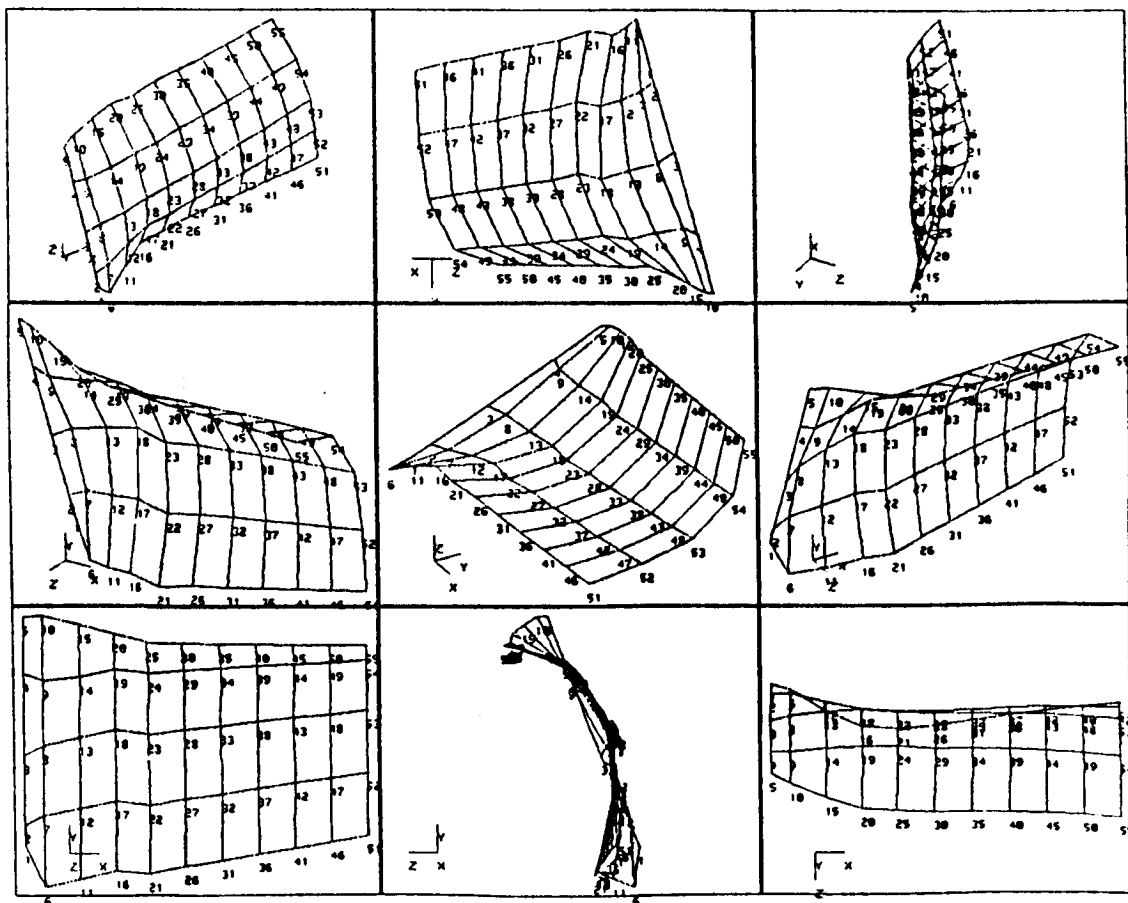
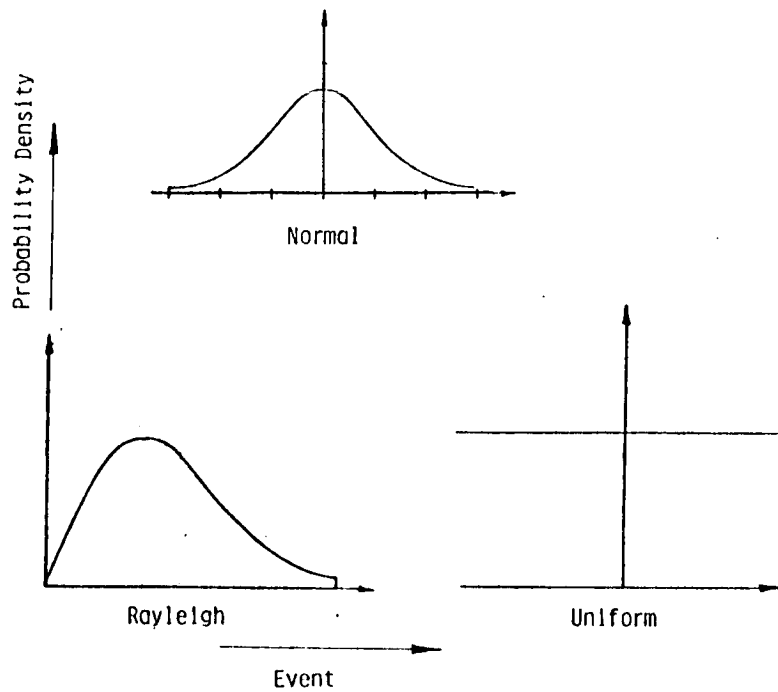
Figure 1.

METHODOLOGY

- Selected random distribution to simulate uncertainties
Normal
- Modelled blade with finite element model
80 Elements
55 Nodes
- Used experiment design to perform simulation
Full factorial design
- Analyzed response
Probabilistic Models
Probabilistic Distributions
Statistical Tests

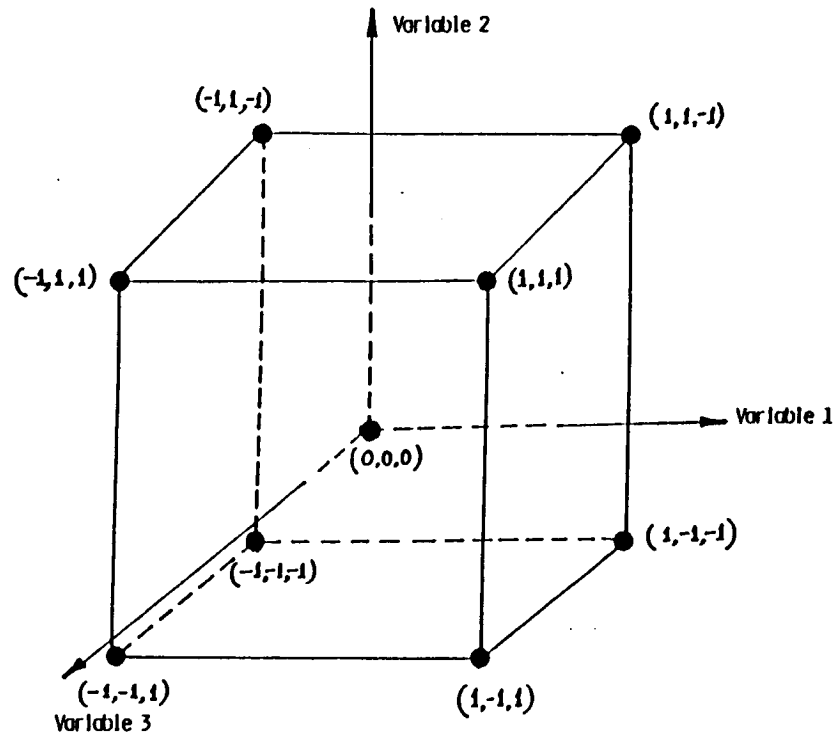
Figure 2.

RANDOM DISTRIBUTIONS



FINITE ELEMENT MODEL OF SSME BLADE

Figure 3.



OPTIMIZED WAY OF STUDYING THREE VARIABLE EFFECTS

FULL FACTORIAL DESIGN

Figure 4.

STRUCTURAL RESPONSE

- NATURAL FREQUENCIES

First

Second

Third

- ROOT (MAXIMUM) STRESS

- TIP DISPLACEMENTS

Figure 5.

TABLE 1 - PROBABILISTIC MODELS - GEOMETRIC PERTURBATIONS

MODEL:

$$\text{DEP. VAR.} = \text{CONSTANT} + \text{COEFF } \mu_1 + \text{COEFF } \mu_2 + \text{COEFF } \mu_3 + \text{COEFF } \sigma_4 + \text{COEFF } \sigma_5 + \text{COEFF } \sigma_6$$

DEPENDENT VARIABLE	CONSTANT	COEFFICIENTS OF					
		μ_1	μ_2	μ_3	σ_4	σ_5	σ_6
FIRST FREQ.	6105.0	-832	-2915	-687	-8362	1897	-9348
SECOND FREQ.	9475.1	-1374	-2961	-408	-11398	-3869	-9095
THIRD FREQ.	15792.	-9434	27592	-7944	-17663	-4899	-44147
ROOT STRESS	63323.	49707	103320	4177	88960	-48191	283960
TIP DISPL.	.00196	.0119	.1287	.0206	-.0132	-.0419	.0067

F tests indicated that all models are good fits.

DISTRIBUTION OF NATURAL FREQUENCIES
- MATERIAL PROPERTIES PERTURBATIONS

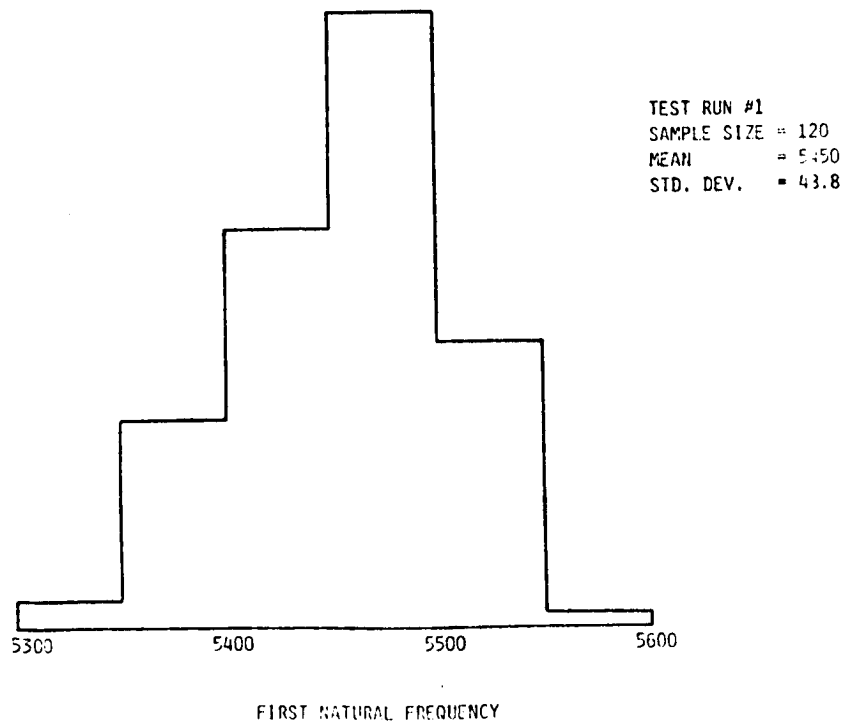


Figure 6.

STATISTICAL TESTS

- t-Test

$$t = \frac{\text{Value} - \text{Mean}}{\text{St. Dev.}}$$

- F-Test

$$F = \frac{(\text{Residual Sum of Squares})/D.F.}{(\text{Regression Sum of Squares})/D.F.}$$

- χ^2 Test

$$\chi^2 = \frac{(\text{Value} - \text{Mean})^2}{\text{Variance}}$$

- Plots

Figure 7.

CONCLUSIONS

- Methodology for SSME Blade has been developed and applied.
- Methodology can be extended to other structural components.
- Geometric uncertainties showed significant effect.
- Material properties uncertainties and their interactions have insignificant effects.
- Range of variation in response quantified.

Figure 8.